Study of Lie Algebras by Using Combinatorial Structures

Manuel Ceballos, Juan Núñez and Ángel F. Tenorio

University of Seville and Pablo de Olavide University

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Contents

Preliminaries

Lie algebra

A Lie algebra $\mathfrak{g}$ is a vector space with a second bilinear composition law ($[,]$) which satisfies: $[X, X] = 0$, $\forall X \in \mathfrak{g}$ and $[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0$, $\forall X, Y, Z \in \mathfrak{g}$.

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Structure constants

A basis $\{e_h\}_{h=1}^{n}$ of $\mathfrak{g}$ is characterized by its **structure constants**: $[e_i, e_j] = \sum c_{i,j}^{h} e_h$, for $1 \leq i, j \leq n$.

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Semisimple and simple Lie algebras

A Lie algebra $\mathfrak{g}$ is **semisimple** if it does not contain any proper abelian ideal. A **simple** Lie algebra is a non-abelian Lie algebra with no non-trivial ideals.

Preliminaries

Upper central series

The upper central series of a Lie algebra $\mathfrak{g}$ is defined as
$\mathcal{C}_1(\mathfrak{g}) = \mathfrak{g},\ \mathcal{C}_2(\mathfrak{g}) = [\mathfrak{g},\mathfrak{g}],\ \mathcal{C}_3(\mathfrak{g}) = [\mathcal{C}_2(\mathfrak{g}),\mathcal{C}_2(\mathfrak{g})],\ \dots,$
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Solvable Lie algebra

If there exists $m \in \mathbb{N}$ such that $\mathcal{C}_m(\mathfrak{g}) \equiv \{0\}$, the Lie algebra $\mathfrak{g}$ is solvable. A solvable Lie algebra is k-step if $\mathcal{C}_k(\mathfrak{g}) \neq \{0\}$ and $\mathcal{C}_{k+1}(\mathfrak{g}) \equiv \{0\}$.

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Nilpotent Lie algebra

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Combinatorial structures and Lie algebras

Let $\mathfrak{g}$ be a n-dimensional Lie algebra with basis $\mathcal{B} = \{e_h\}_{h=1}^{n}$. The pair $(\mathfrak{g}, \mathcal{B})$ can be associated with a combinatorial structure:

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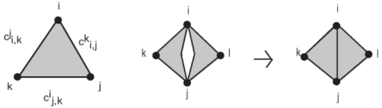
    - If $c_{i,j}^k = c_{j,k}^i = c_{i,k}^j = 0$, the triangle is not drawn.
    - If a structure constant is null, the corresponding edge is drawn with a discontinuous line (*ghost edge*). For the *degree* of a vertex both types are assumed.
    - If two triangles of vertices i, j, k, and i, j, l satisfy $c_{i,j}^k = c_{i,j}^l$, the edge ij is shared.

Combinatorial structures and Lie algebras



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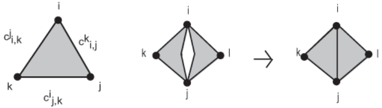

- **Given two vertices $i < j$, if $c^i_{i,j} \neq 0$ or $c^j_{i,j} \neq 0$, then a directed edge is drawn.**

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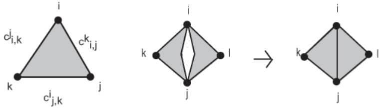

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Combinatorial structures and Lie algebras

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A vertex v is said to be a going-in (respectively going-out) vertex if all the directed incident edges with v are oriented towards v (respectively, from v).

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Every Lie algebra with a selected basis is associated with a combinatorial structure. This association depends on the selected basis.

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Complete graph

A cycle digraph, G, is defined as a cycle graph with directed edges.

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Given a combinatorial structure, T, of n vertices:

- Label all the vertices by $1, 2, \ldots, n$, following the positive counterclockwise.

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Given a combinatorial structure, T, of n vertices:

- Label all the vertices by $1, 2, \ldots, n$, following the positive counterclockwise.
- The weight of the edge ij will be denoted by $c_{i,j}$.
- Define a vector space V with basis $\{e_1, \ldots, e_n\}$ where e_i corresponds to the vertex i of T and brackets

$$[e_i, e_j] = c_{i,j}^i e_i + c_{i,j}^j e_j.$$

Case $n = 3$

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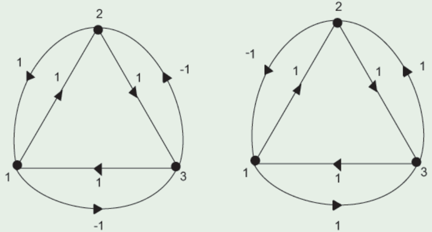

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$$c_{j,k}^{j} c_{i,j}^{i} + c_{j,k}^{k} c_{i,k}^{i} = 0, \ c_{i,k}^{k} c_{j,k}^{j} - c_{i,k}^{i} c_{i,j}^{j} = 0, \ c_{i,j}^{i} c_{i,k}^{k} + c_{i,j}^{j} c_{j,k}^{k} = 0.$$

When imposing all the Jacobi identities and the restrictions: $c_{p,p+1}^{p+1} = 1$, for all $p \in \{1, \ldots, n\}$ and $c_{1,n}^{1} = 1$, we obtain the law of a particular Lie algebra, that can be considered over $Z/3Z$.

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$$\left.\begin{array}{l} [e_p, e_q] = -e_p + e_q \\ [e_p, e_n] = e_p + e_n, \end{array}\right\} \quad \text{where} \left\{\begin{array}{l} 1 \leq p \leq n-2; \\ p+1 \leq q \leq n-1. \end{array}\right.$$

In this way, we can establish the following

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Proposition

Let us consider a well-oriented, weighted cycle digraph G with double edges of 3 vertices. Then, G is associated with a 3-dimensional Lie algebra if and only if the weights of its edges satisfy one of the following constraints

(i) $c_{1,3}^1 = 1$, $c_{1,2}^2 = 1$, $c_{2,3}^3 = 1$, $c_{2,3}^2 = 1$, $c_{1,3}^3 = 1$ and $c_{1,2}^1 = -1$. In this case, the Lie algebra, denoted by $\mathfrak{g}$, is perfect.

(ii) $c_{1,3}^1 = 1$, $c_{1,2}^2 = 1$, $c_{2,3}^3 = 1$, $c_{2,3}^2 = -1$, $c_{1,3}^3 = -1$ and $c_{1,2}^1 = 1$. In this case, the Lie algebra, denoted by $\mathfrak{h}$, is 2-step solvable and non-nilpotent.

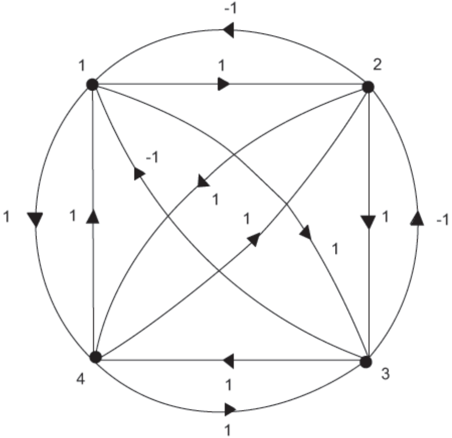
Proposition

Let us consider a well-oriented, weighted cycle digraph G with double edges of $n \geq 4$ vertices. Then, G is associated with an n-dimensional Lie algebra if and only if the weights of the edges satisfy

$$c_{p,q}^{p} = -1, \; c_{p,q}^{q} = 1, \; c_{p,n}^{p} = c_{p,n}^{n} = 1, \text{ where } \begin{cases} 1 \leq p \leq n-2; \\ p+1 \leq q \leq n-1. \end{cases}$$

The Lie algebra associated with the digraph is unique and is denoted by $\mathfrak{g}_n$. Moreover, the Lie algebra $\mathfrak{g}_n$ is 2-step solvable and non-nilpotent.

Next, for the sake of example, we show the digraph associated with a 4-dimensional Lie algebra.



Implementing the algorithm with Maple

We implement the algorithm by using the symbolic computation package MAPLE with a subroutine and a main routine.

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Subroutine subr

```
> subr:=proc(n,i,j,k)
> S:=c[j,k,j]*c[i,j,i]+c[j,k,k]*c[i,k,i]=0,
> c[i,k,k]*c[j,k,j]-c[i,k,i]*c[i,j,j]=0,
> c[i,j,i]*c[i,k,k]+c[i,j,j]*c[j,k,k]=0;
> for q from 1 to n-1 do
> S:=op(S),c[q,q+1,q+1]=1;end do;
> S:=op(S),c[1,n,1]=1;
> return S; end proc:
```

Implementing the algorithm with Maple

Routine mainr

```
> mainr:=proc(n)
> local L,T;
> L:=choose(n,3); T:=;
> for p from 1 to nops(L) do
> T:=op(T),op(subr(n,L[p][1],L[p][2],L[p][3]));
> end do;
> return solve(T);
> end proc;
```

We have proved that several digraphs are associated with 2-step solvable non-nilpotent Lie algebras under some restrictions. Every 2-step solvable non-nilpotent Lie algebra is associated with a digraph?

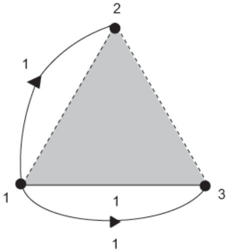
$\mathfrak{g}$	Lie brackets	Parameters
$\mathfrak{r}_2$	$[e_1, e_2] = e_2$	
$\mathfrak{r}_3$	$[e_1, e_2] = e_2, [e_1, e_3] = e_2 + e_3$	
$\mathfrak{r}_{3,p}$	$[e_1, e_2] = e_2, [e_1, e_3] = p e_3$	$p \in \mathbb{C}^*, \lvert p \leq 1$
$\mathfrak{r}_{4,q}$	$[e_1, e_2] = e_2, [e_1, e_3] = e_3, [e_1, e_4] = q e_4$	$q \in \mathbb{C}^*$
$\mathfrak{r}_{4,\alpha,\beta}$	$[e_1, e_2] = e_3, [e_1, e_3] = e_4, [e_1, e_4] = \alpha e_2 - \beta e_3 + e_4$	$\alpha \in \mathbb{C}^*$, $\beta \in \mathbb{C}$ or $\alpha, \beta = 0$
$\mathfrak{g}_{4,11}^{\alpha}$	$[e_1, e_2] = e_3, [e_1, e_3] = e_4, [e_1, e_4] = \alpha(e_2 + e_3)$	$\alpha \in \mathbb{C}^*$
$\mathfrak{g}_{4,12}$	$[e_1, e_2] = e_3, [e_1, e_3] = e_4, [e_1, e_4] = e_2$	

$\mathfrak{g}$	Lie brackets	Parameters
$\mathfrak{g}_{4,13}$	$[e_1, e_2] = \frac{1}{3} e_2 + e_3, [e_1, e_3] = \frac{1}{3} e_3, [e_1, e_4] = \frac{1}{3} e_4$	
$\mathfrak{g}_{5,7}^{p,q,r}$	$[e_1, e_5] = e_1, [e_2, e_5] = pe_2$ $[e_3, e_5] = qe_3, [e_4, e_5] = re_4$	$pqr \neq 0$ $-1 \leq r \leq q \leq r \leq 1$
$\mathfrak{g}_{5,8}^{\gamma}$	$[e_2, e_5] = e_1, [e_3, e_5] = e_3, [e_4, e_5] = \gamma e_4,$	$0 < \lvert\gamma\rvert \leq 1$
$\mathfrak{g}_{5,9}^{\beta,\gamma}$	$[e_1, e_5] = e_1, [e_2, e_5] = e_1 + e_3$ $[e_3, e_5] = \beta e_3, [e_4, e_5] = \gamma e_4$	$0 \neq \gamma \leq \beta$
$\mathfrak{g}_{5,10}$	$[e_2, e_5] = e_1, [e_3, e_5] = e_2, [e_4, e_5] = e_4$	
$\mathfrak{g}_{5,11}^{\gamma}$	$[e_1, e_5] = e_1, [e_2, e_5] = e_1 + e_2$ $[e_3, e_5] = e_2 + e_3, [e_4, e_5] = \gamma e_4$	$\gamma \neq 0$
$\mathfrak{g}_{5,12}$	$[e_1, e_5] = e_1, [e_2, e_5] = e_1 + e_2$ $[e_3, e_5] = e_2 + e_3, [e_4, e_5] = e_3 + e_4$	

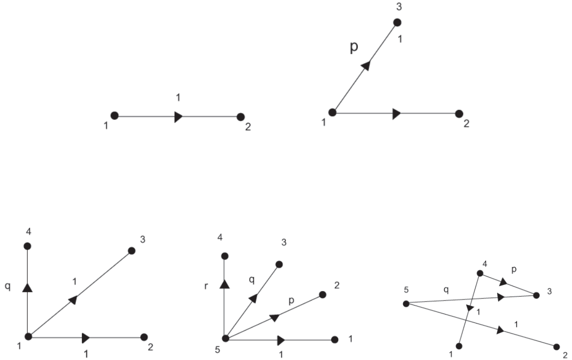
$\mathfrak{g}$	Lie brackets	Parameters
$\mathfrak{g}_{5,13}^{p,s,\gamma}$	$[e_1, e_5] = e_1, [e_2, e_5] = \gamma e_2$ $[e_3, e_5] = pe_3 - se_4, [e_4, e_5] = se_3 + pe_4$	$\gamma s \neq 0, \lvert\gamma\rvert \leq 1$
$\mathfrak{g}_{5,14}$	$[e_2, e_5] = e_1, [e_3, e_5] = pe_3 - e_4$ $[e_4, e_5] = e_3 + pe_4$	
$\mathfrak{g}_{5,15}^{\gamma}$	$[e_1, e_5] = e_1, [e_2, e_5] = e_1 + e_2$ $[e_3, e_5] = \gamma e_3, [e_4, e_5] = e_3 + \gamma e_4$	$-1 \leq \gamma \leq 1$
$\mathfrak{g}_{5,16}^{p,s}$	$[e_1, e_5] = e_1, [e_2, e_5] = e_1 + e_2$ $[e_3, e_5] = pe_3 - se_4, [e_4, e_5] = se_3 + pe_4$	$s \neq 0$
$\mathfrak{g}_{5,17}^{p,q,s}$	$[e_1, e_5] = pe_1 - e_2, [e_2, e_5] = e_1 + pe_2$ $[e_3, e_5] = qe_3 - se_4, [e_4, e_5] = se_3 + qe_4$	$s \neq 0$
$\mathfrak{g}_{5,18}^{p}$	$[e_3, e_5] = e_1 + pe_3 - e_4, [e_2, e_5] = e_1 + pe_2$ $[e_1, e_5] = pe_1 - e_2, [e_4, e_5] = e_2 + e_3 - pe_4$	$p \geq 0$
$\mathfrak{g}_{5,22}$	$[e_2, e_3] = e_1, [e_2, e_5] = e_3, [e_4, e_5] = e_4$	

$\mathfrak{g}$	Lie brackets	Parameters
$\mathfrak{g}_{5,27}$	$[e_2, e_3] = e_1, [e_1, e_5] = e_1$ $[e_3, e_5] = e_3 + e_4, [e_4, e_5] = e_1 + e_4$	
$\mathfrak{g}_{5,29}$	$[e_2, e_3] = e_1, [e_1, e_5] = e_1, [e_2, e_5] = e_2, [e_3, e_5] = e_4$	
$\mathfrak{g}_{5,32}^{h}$	$[e_2, e_4] = e_1, [e_3, e_4] = e_2, [e_1, e_5] = e_1$ $[e_2, e_5] = e_2, [e_3, e_5] = he_1 + e_3$	
$\mathfrak{g}_{5,33}^{p,q}$	$[e_1, e_4] = e_1, [e_3, e_4] = pe_3, [e_2, e_5] = e_2, [e_3, e_5] = qe_3$	$p^2 + q^2 \neq 0$
$\mathfrak{g}_{5,34}^{\alpha}$	$[e_1, e_4] = \alpha e_1, [e_2, e_4] = e_2, [e_3, e_4] = e_3$ $[e_1, e_5] = e_1, [e_3, e_5] = e_2$	
$\mathfrak{g}_{5,35}^{h,\alpha}$	$[e_1, e_4] = he_1, [e_2, e_4] = e_2, [e_3, e_4] = e_3$ $[e_1, e_5] = \alpha e_1, [e_2, e_5] = -e_3 [e_3, e_5] = e_2$	$h^2 + \alpha^2 \neq 0$
$\mathfrak{g}_{5,37}$	$[e_1, e_4] = e_1, [e_2, e_4] = e_2 [e_1, e_5] = -e_2,$ $[e_2, e_5] = e_1, [e_4, e_5] = e_3$	
$\mathfrak{g}_{5,38}$	$[e_1, e_4] = e_1, [e_2, e_5] = e_2, [e_4, e_5] = e_3$	

Let us note that for example the combinatorial structure associated with the Lie algebra $\mathfrak{r}_3$ is not a digraph since it contains a full triangle.



The following figures show the digraphs associated with 2-step solvable non-nilpotent Lie algebras up to dimension 5. These algebras are: $\mathfrak{r}_2$, $\mathfrak{r}_{3,p}$, $\mathfrak{r}_{4,q}$, $\mathfrak{g}_{5,7}^{p,q,r}$ and $\mathfrak{g}_{5,33}^{p,q}$.



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Manuel Ceballos, Juan Núñez and Ángel F. Tenorio

University of Seville and Pablo de Olavide University

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