Applied Linear Algebra

In honor of Hans Schneider

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Reviewers:

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Honorable Lecture

ON COMMUTING MATRICES IN MAX ALGEBRA AND IN CLASSICAL NONNEGATIVE ALGEBRA

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This research is joint with R. Katz and S. Sergeev. We study commuting matrices in max algebra and nonnegative linear algebra. Our starting point is the existence of a common eigenvector which directly leads to max analogues of some classical results for complex matrices . We also investigate Frobenius normal forms of commuting matrices, particularly when the Perron roots of the components are distinct. For the case of max algebra, we show how the intersection of eigencones of commuting matrices may be described. We also consider connections with Boolean algebra which enables us to prove that two commuting irreducible matrices in max algebra have a common eigennode.

Plenary Lectures

TOTALLY NONNEGATIVE $(0,1)$-MATRICES [1]

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We consider $(0,1)$-matrices all of whose eigenvalues are nonnegative real numbers. The nonnegativity of eigenvalues is guaranteed if the matrix is totally nonnegative (TNN), but a $(0,1)$-matrix can have all nonnegative eigenvalues but not be TNN. TNN (0,1)-matrices with no zero rows or columns are characterized by four forbidden submatrices of orders 2 and 3. The maximum number of 0s in an irreducible, totally nonnegative $(0,1)$-matrix of order n is $(n-2)^2$, and those matrices with this number of 0s can be characterized. The minimum Perron value of an irreducible, totally nonnegative $(0,1)$-matrix of order n equals $2 + 2\cos\left(\frac{2\pi}{n+2}\right)$ and the matrices with this Perron value can be characterized.

[1] Joint work with Steve Kirkland

MAX-LINEAR SYSTEMS: FEASIBILITY AND REACHABILITY

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Max-algebra provides mathematical theory and techniques for solving non-linear problems that can be given the form of linear problems, when arithmetical addition is replaced by the operation of maximum and arithmetical multiplication is replaced by addition. Problems of this kind are sometimes of a managerial nature, arising in areas such as manufacturing, transportation, allocation of resources and information processing technology. Another setting, algebraically equivalent to the first one, is obtained when addition is replaced by the maximum and arithmetical multiplication is unchanged but the ground set is restricted to non-negative reals. Two classes of problems in max-algebra attract particular attention: feasibility (one or two-sided systems of linear equations, the eigenproblem and generalized eigenproblem) and reachability (reachability of eigenspaces by matrix orbits and robustness of matrices). In the first part of the talk we show that all these problems have an interpretation based on multi-processor interactive systems (MPIS), in which processors work in stages and the starting times of the machines at each stage are determined solely by the work of other machines at the previous stage. In the second part we pay a more detailed attention to robustness of matrices and the generalized eigenproblem. We present an efficient criterion for deciding whether a matrix is robust, which is based on the analogue of the Perron-Frobenius spectral theory in max-algebra. In contrast, the generalized eigenproblem seems to be one of the most intractable problems in max-algebra. We present a method for narrowing the search for generalized eigenvalues. This method is polynomial and in some cases finds all such eigenvalues.

GRAPH SPECTRA IN COMPUTER SCIENCE[2]

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In this paper we shall give a survey of applications of the theory of graph spectra to computer science. Eigenvalues and eigenvectors of several graph matrices appear in numerous papers on various subjects relevant to information and communication technologies. In particular, we survey applications in modelling and searching Internet, in computer vision, data mining, multiprocessor systems, statistical databases, and in several other areas. Some related new mathematical results are included together with several comments on perspectives for future research.

[2] Joint work with Slobodan Simić and Dragan Stevanović

COMBINED MATRICES OF MATRICES IN SPECIAL CLASSES

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We call *combined matrix* $C(A)$ of nonsingular matrix A the Hadamard product $A \circ (A^T)^{-1}$. It is well known basic properties are: All row- and column-sums are equal to one, and independence of multiplication of A by nonsingular diagonal matrices from either side.

In particular, the sequence of diagonal entries of $C(A)$ seems to have interesting properties (in a sense, reminding of condition numbers). We shall report on properties of this sequence for the case that A belongs to some special class, such as A positive definite, A an M-matrix, A totally positive, A tridiagonal, A complementary basic (in the sense of the previous paper of the author), etc.

COMPUTING GUARANTEED ERROR BOUNDS FOR SOLUTIONS OF MATRIX EQUATIONS

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We consider linear matrix equations like the Sylvester or the Lyapunov equation as well as the quadratic equation $X^2 = A$ defining the matrix square root. Usually, an approximate solution $\tilde{X}$ to such a linear or nonlinear matrix equation is computed using an efficient algorithm carried out in floating point arithmetic. In this talk we address the issue of how to obtain reliable error bounds, individually for each entry of $\tilde{X}$. An established approach is to consider an associated fixed point problem for which the hypothesis of Brouwer's fixed point theorem is then checked computationally using interval arithmetic. Since a correct implementation of interval arithmetic, as it is realized, for example, in the Matlab toolbox Intlab, takes rounding errors into account, these methods obtain error bounds which are correct with mathematical certainty.

Applying the standard interval techniques for matrix equations yields a computational complexity of $\mathcal{O}(n^6)$, which is far too high. We therefore discuss modified methods for which the complexity is reduced to $\mathcal{O}(n^3)$. Since, moreover, the by far most costly part of the computations can be formulated as matrix-matrix operations, these modified methods can be implemented very efficiently in Intlab. As a result, the computation of the guaranteed error bounds takes a time which is comparable to that for the computation of the approximate solution. We will illustrate this with several numerical experiments.

The conceptual breakthrough behind the Lokta-Volterra stability model

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This talk will review the conceptual breakthrough that lead to the Lokta Volterra theory, solving various stability problems which are modeled by Lyapunov diagonal stability and other forms of matrix stability.

LINEAR ALGEBRA AND QUANTUM COMPUTING

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The mathematical framework of quantum mechanics is Hilbert space theory. As a result, linear algebra techniques are useful in the study of problems in quantum computing. In this talk, we will give a brief introduction to quantum computing, and then describe results and problems in linear algebra related to quantum computing.

MONOTONE SOLUTIONS OF SOME LINEAR EQUATIONS IN BANACH SPACES

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In this contribution motivated by analysis concerning bounds of topological entropy as studied in [1] it is shown that a well known sufficient condition for a difference and ordinary differential equation with constant real coefficients to possess strictly monotone solution appears to be also necessary. Transparent proofs of adequate generalizations to Banach space analogs are presented.

In the context of Linear Algebra the following Problem can serve as appropriate demonstration.

0.1 Problem *Let* $N \geq 1$ *and*

$$x_{k+N} = a_1 x_k + a_2 x_{k+1} + ... + a_N x_{k+N-1},\ k = 0, 1, ..., \tag{1}$$

where $a_1, ..., a_N$ *are arbitrary real numbers. Find necessary and sufficient conditions guaranteeing existence of a strictly monotone solution of the above given difference equation.*

A solution to the Problem 0.1 has appeared as a very useful tool in developing a theory characterizing the complexity of some classes of functions via the topological entropy [1]. In order to give to the audience a flavour of the theory mentioned some details needed for this purpose will be shown.

References

[1] Bobok J. *On entropy of patterns given by interval maps.* Fundamenta Mathematicae **162**(1999), 1-36.

STABILITY ANALYSIS OF DIFFERENTIAL ALGEBRAIC SYSTEMS [3]

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Lyapunov and exponential dichotomy spectral theory is extended from ordinary differential equations (ODEs) to nonautonomous differential-algebraic equations (DAEs). By using orthogonal changes of variables, the original DAE system is transformed into appropriate condensed forms, for which concepts such as Lyapunov exponents, Bohl exponents, exponential dichotomy and spectral intervals of various kinds can be analyzed. Some essential differences between the spectral theory for ODEs and that for DAEs are pointed out. It is also discussed how numerical methods for computing the spectral intervals associated with Lyapunov and Sacker-Sell (exponential dichotomy) can be extended.

Some numerical examples are presented to illustrate the theoretical results.

[3] Joint work with Vu Hoang Linh

THE GROUP AND DRAZIN GENERALIZED INVERSES OF SINGULAR M–MATRICES AND THEIR APPLICATIONS[4]

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At the ILAS 5th Conference on Linear Algebra in Atlanta, 1995, I gave a talk by the same title. I did not expect then to be still working on properties and applications of the group and Drazin generalized inverses of M–Matrices 15 years later. If anything, I believe that the usage and interest in generalized inverses has widened in these years and so has the specific interest in the group and Drazin generalized inverses of M–matrices.

At the Atlanta conference I spoke about applications to:

- Iterative methods for singular system.
- Pertubation theory of the Perron root and vector of a nonnegative matrix.
- The computation of nonnegative bases for the Perron eigenspace of a nonnegative matrix.
- The algebraic connectivity of undirected graphs.

[4]The results come from joint works with several people. Among them Professors Minerva Catral, Steve Kirkland, Jason Molitierno, Bryan Shader, Raymond Sze, and Jianhong Xu.

- Markov chains.

I will mention results since then on all the above listed areas, but I will particularly concentrate on results concerning applications of group generalized inverses of M–matrices to Markov chains. Special attention will be paid to the inverse mean passage matrix problem and its relationship to the inverse M–matrix problem.

EIGENVALUE LOCALIZATION AND SOME CLASSES OF MATRICES RELATED TO POSITIVITY

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Recent results on eigenvalue localization are presented. The relationship with some structured classes of matrices related to positivity is analyzed. In particular, motivated by eigenvalue localization problems, new classes of matrices related to positivity have been introduced recently. We illustrate with new results that nonsingular classes of matrices lead to eigenvalue localization results and that these localization results are in turn more useful when applied to special classes of matrices are illustrated with new results. New results on the localization of the smallest eigenvalue of certain classes of matrices related to positivity are presented. Applications and related problems are also considered.

PARTITION PROBLEMS: OPTIMALITY AND CLUSTERING

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Partition problems constitute a large class of combinatorial optimization problems. Of particular interest are problems where it ispossible to restrict attention to solutions that exhibit clustering properties, facilitating the solution of the partition problem in polynomial time. The talk will introduce a classification of partition problem and survey of numerous approaches to solve such problems by focusing on partitions that exhibit clustering properties. The main technique concern the study of vertices of corresponding partition polytopes.

THE FIELD OF VALUES OF OBLIQUE PROJECTIONS[5]

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We highlight some properties of the field of values (or numerical range) $W(P)$ of an oblique projector P on a Hilbert space, i.e., of an operator satisfying $P^2 = P$. If P is neither null nor the identity, we present a direct proof showing that $W(P) = W(I - P)$, i.e., the field of values of an oblique projection coincides with that of its complementary projection. We also show that $W(P)$ is an elliptical disk (i.e., the set of points circumscribed by an ellipse) with foci at 0 and 1 and eccentricity $1/\|P\|$. These two results combined provide a new proof of the identity $\|P\| = \|I - P\|$. We discuss the relation between the minimal canonical angle between the range and the null space of P and the shape of $W(P)$.

[5] Joint work with Valeria Simoncini

PERRON-FROBENIUS THEORY AND POSITIVITY IN LINEAR ALGEBRA

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We will first take a retrospective look at the celebrated Perron-Frobenius Theorem and survey its diverse manifestations in other fields of mathematics. We will then focus on matrix classes inspired by the Perron-Frobenius theory and take time to describe challenging problems regarding these matrices.

AN APPLICATION OF THE PERRON-FROBENIUS THEORY OF NONNEGATIVE MATRICES TO THE SYNCHRONIZATION OF CHAOTIC OSCILLATIONS[6]

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Let k be a fixed positive integer, let m be any positive integer with $m \geq k+1$, and let $B = [b_{i,j}] \in \mathbb{R}^{m,m}$ satisfy

$$\begin{cases} b_{i,i} = 0, & (\text{all } 1 \leq i \leq m), \\ b_{i,j} = 0 \,\text{or}\, 1, & (\text{all}), i \neq j,\ 1 \leq i, j \leq m)), \text{ and} \\ \sum_{j=1}^{m} b_{i,j} = k, & (\text{all } 1 \leq i \leq m). \end{cases} \tag{2}$$

Then, B is nonnegative matrix in $\mathbb{R}^{m,m}$, which is easily seen to have a spectral radius $\rho(B)$ such $\rho(B) = k$. If $\mathcal{B}(k)$ denotes the collection of the spectra, $\sigma(B)$, for all B satisfying (**??**), we show that

$$\overline{\bigcup_{B \in \mathcal{B}(1)} \sigma(B)} = \{0\} \cup \{z \in \mathbb{C} : |z| = 1\}, \text{ for } k = 1, \text{and} \tag{3}$$

$$\overline{\bigcup_{B \in \mathcal{B}(k)} \sigma(B)} = \{z \in \mathbb{C} : |z| \leq k\}, \text{ for each } k \geq 2. \tag{4}$$

In addition, we present numerical results, from some very large Matlab program, which indicates the validity of (**??**) and (**??**).

[6] Joint work with Alessandro Rizzo

Invited Lectures

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CHARACTERIZATION OF $\alpha 1$ AND $\alpha 2$-MATRICES [7]

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This talk deals with some properties of α-matrices, which are subclasses of invertible H-matrices. In particular, new characterizations of $\alpha 1$ and of $\alpha 2$-matrices are given. Considering these characterizations some algebraic properties of these matrices are studied. Then, we apply the above characterizations to derive a new eigenvalue inclusion set associated with the class of α-matrices.

[7] Joint work with Ljiljana Cvetković, Vladimir Kostić and Francisco Pedroche

NEWTON'S INEQUALITIES ON A LATTICE

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If A is a square matrix of order n and $\alpha \subseteq \{1, 2, ..n\}$ then $A[\alpha]$ will denote the sub matrix of A contained in the intersection of the rows and columns indexed by α. Newton's inequalities have been shown to hold for Hermitian and for M Matrices by Olga Holtz. If A is either a non-negative definite Hermitian matrix or a M Matrix and $\beta \subseteq \{1, 2, ..n\}$then the determinant of A satisfies: $\det A[\alpha] \det A[\beta] \geq \det A[\alpha \cap \beta] \det A[\alpha \cup \beta]$. If F is a real valued function then for $1 \leq k \leq n$ define $S_k^F = \frac{1}{\binom{n}{k}} \sum_{\alpha \subseteq \{1,2,..n\}, k=|\alpha|} F(\alpha)$ and $S_0^F = 1$. We will show that if F satisfies $F(\alpha \cap \beta)F(\alpha \cup \beta) \leq F(\alpha)F(\beta)$ for all α and β that are subsets of $\{1, 2, ..n\}$ with $F(\phi) = 1$ then $(S_k^F)^2 \geq S_{k-1}^F S_{k+1}^F$.

SOME LINEAR ALGEBRA TOOLS FOR INFORMATION RETRIEVAL[8]

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Internet Algorithmics and Information Retrieval (IR) have become grand challenge application areas for Numerical Linear Algebra. Important problems include dimensionality reduction of very large datasets, and ranking webpages based on link analysis of the Web graph. In this presentation we review some problems from this area, and focus on Nonnegative Matrix Factorization (NMF). As a data analysis tool, NMF appears to offer superior intepretability over standard dimensionality reduction algorithms such as the SVD but unfortunately it is much harder to compute. We discuss recent approaches to NMF-type computatioons for general nonnegative as well as nonnegative symmetric matrices. We also describe relevant features of the current version of TMG, a MATLAB toolbox for the rapid prototyping of algorithms and dataset development for Information Retrieval applications.

[8]Joint work with V. Kalofolias and D. Zeimpekis

A LOOK-BACK TECHNIQUE OF RESTART FOR THE GMRES(n) METHOD[9]

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In recent years, the several Krylov subspace methods have been proposed for large, sparse, and nonsymmetric linear systems $Ax = b$. The GMRES method and the BiCGSTAB method (and its variants) are well-known as the most successful Krylov subspace methods for the systems.

In this talk, we consider the GMRES method proposed by Saad and Schultz. The GMRES method shows a good convergence; however, it has some difficulties in storage and computational costs. Therefore the so-called *restart* is generally used as a practical choice. The restarted version of the GMRES method is called the GMRES(m) method.

In each restart of the GMRES(m) method, the initial guess is usually fixed on the approximate solution obtained at previous restart. In this talk, we focus on this fixed initial guess, and then, we are motivated by this question "if let the initial guess of each restart of the GMRES(m) method be free, what happens?"

Our main goal of this talk is to answer this question and to propose a Look-Back technique of restart for the GMRES(m) method. Numerical results will show a potentiality for efficient convergence

[9] Joint work with Tomohiro Sogabe and Shao-Liang Zhang

within the proposed method compared with the GMRES(m) method.

GERŠGORIN-TYPE LOCALIZATIONS FOR GENERALIZED EIGENVALUES AND THEIR APPROXIMATIONS[10]

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Localizations of generalized eigenvalues through the Geršgorin set, introduced by Stewart in [1], have been recently rediscovered and discussed in [2]. Here, we will present new localization sets of Geršgorin-type (Brauer sets, Brualdi sets, CKV sets, etc.) for generalized eigenvalues, and discuss the ways in which they can be approximated. Using a concept of the generalized diagonal dominance, we will prove that the obtained approximation have some of the basic properties of the original Geršgorin-type sets which makes them a handy tool for the localization of generalized eigenvalues. Especially, practical conditions under which Geršgorin set for generalized eigenvalues can be well approximated by simple circles will be given, together with similar results for the Geršgorin-type sets.

References

[1] G. W. Stewart, *Geršgorin theory for generalized eigenvalue problem* $ax = \lambda x$, *Math. Comput.* **29** (1975), 600–606.

[2] V. Kostić, Lj. Cvetković and R. S. Varga, *Geršgorin-type localizations of generalized eigenvalues*, *Numerical Linear Algebra with Applications* **16** (2009), no. 11-12, 883–898.

[10] Joint work with Richard S. Varga and Ljiljana Cvetković

CONVERGENCE OF SKEW-SYMMETRIC ITERATIVE METHODS [11]

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Field of values for matrix has been exploited for investigation of convergence for triangular (TIM) and product triangular (PTIM) skew-symmetric iterative methods which was introduced by first co-author. Formulaes for connection between field of values for initial matrix, matrix which determinate iterative method and eigenvalues of iterative matrix have been obtained. It was shown that TIM and PTIM can converge in case when initial matrix isn't positive real.

[11] This work was supported by RFBR, grant N09-01-00023a
Joint work with B.L.Krukier

THE UPPER BOUND FOR THE EXPONENT OF A PRIMITIVE, CONE PRESERVING MATRIX [12]

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Let K be a proper cone in $\mathbb{R}^n$. We consider $A \in \mathbb{R}^{n\times n}$ which is K-*primitive*, that is, there exists a positive integer l such that $A^l x \in \text{Int}K$ for every $0 \neq x \in K$. The smallest such l is called the *exponent* of A, denoted by $\gamma(A)$.

We consider an upper bound for $\gamma(A)$ in terms of n and m, the number of extreme rays of A. We show that $\gamma(A) \leq (m-1)(n-1)+1$. This upper bound reduces to Wielandt's well known bound provided that m.

The sharpness of the upper bound is considered, as well as (under some additional assumptions) uniqueness of K and A for which the upper bound is attained.

[12] Joint work with Bit-Shun Tam

LOEWNER MATRIX ORDERING IN ESTIMATION OF THE SMALLEST SINGULAR VALUES[13]

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The importance of singular values in many applications is well-known. In particular, lower bounds for the smallest singular value of a nonsingular matrix are very useful in many fields. There is a lot of literature on this subject. Let us mention the lower bound derived by Varah [A lower bound for the smallest singular value of a matrix. Linear Algebra Appl. 11 (1975), 3-5] for matrices that are simultaneously strictly diagonally dominant by rows and columns. Related bounds for more general classes of matrices were obtained in [R.S. Varga On diagonal dominance arguments for bounding $\|A^{-1}\|_\infty$. Linear Algebra Appl. 14 (1976), 211-217]. Gerschgorin Theorem was used in Theorem 2 of [C.R. Johnson, A Gersgorin-type lower bound for the smallest singular value. Linear Algebra Appl. 112 (1989), 1–7] to derive bounds for the smallest singular value, and more general theorems were used to obtain the bounds in [C.R. Johnson and T. Szulc, Further lower bounds for the smallest singular value. Linear Algebra Appl. 272 (1998), 169–179]. See also [C.R. Johnson, T. Szulc and D. Wojtera-Tyrakowska, Optimal Gersgorin-style estimation of extremal singular values. Linear Algebra Appl. 402 (2005), 46–60].

Our way to get simple bounds for the smallest singular value is to make use of the known fact that the smallest singular value of a

[13] The results are joint ones with J.M. Peña

matrix A is greater than or equal to the minimal eigenvalue of the Hermitian part $H(A)$ of A and focus on Hermitian matrices which are, in the sense of Loewner matrix ordering, below of $H(A)$. We develop two different applications of this ordering to the problem of bounding the smallest singular value. These applications are complementary and require a computational cost of $\mathcal{O}(n^2)$ elementary operations to bound the smallest singular value of an $n \times n$ matrix.

We show that these bounds are sharper than those of [C.R. Johnson and T. Szulc, Further lower bounds for the smallest singular value. Linear Algebra Appl. 272 (1998), 169–179] for there considered matrices.

GAMM ANLA Workshop

SINGULAR VALUE AND NORM INEQUALITIES FOR OPERATORS [14]

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A general singular value inequality for sums and products of Hilbert space operators is given. This inequality generalizes several recent singular value inequalities, and includes that if A, B and X are positive operators on a complex Hilbert space H, then

$$s_j(A^{(1/2)}XB^{(1/2)}) \;\; (1/2) \;\; X \;\; s_j(A+B), j = 1, 2, ...$$

Related singular value inequalities for sums and products of operators are also presented.

[14] Joint work with Khalid Shebrawi

POSITIVITY PRESERVING DISCRETIZATIONS FOR DIFFERENTIAL-ALGEBRAIC EQUATIONS [15]

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In this talk we will present positivity preserving discretizations of dynamical systems with constraints.
We give a positivity characterization of these systems that extends the corresponding results for ordinary differential equations about suitable positivity conditions for the algebraic components.
With this characterization, we derive conditions under which this property is preserved in the process of numerical integration, i.e. under which assumptions we can establish a positive, discrete approximation of our original system.
As an example, we will consider the positivity of one- and multistep methods that can be applied to Differential-Algebraic Equations without order reduction.

[15] Joint work with Volker Mehrmann

A NEWTON-GALERKIN-ADI METHOD FOR LARGE-SCALE ALGEBRAIC RICCATI EQUATIONS[16]

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The numerical solution of large-scale algebraic Riccati equations (AREs) is the central task in solving optimal control problems for linear and, using receding-horizon techniques, also nonlinear instationary partial differential equations. Large-scale AREs also occur in several model reduction methods for dynamical systems. Due to sparsity and the large dimensions of the resulting coefficient matrices, standard eigensolver-based methods for AREs are not applicable in this context. In the recent two decades, several approaches for such large-scale AREs have been suggested. They mainly fall into two categories: 1.) (Galerkin-)project the solution onto a low-dimensional subspace, e.g., a suitable Krylov subspace, solve the small scale ARE with a standard solver, and prolongate the solution to full-scale; 2.) employ Newton's method and exploit sparsity in the resulting linear system of equations (= a Lyapunov equation) to be solved in each step. We will present a hybrid method based on exploiting the advantages of both ideas. Numerical experiments confirm the high efficiency of this new method and demonstrate its applicability to the aforementioned application areas.

[16] Joint work with Jens Saak

USING STRUCTURE FOR PRECONDITIONING SYSTEMS ARISING FROM THE KKR GREEN FUNCTION METHOD

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Recently a linear scaling method for the calculation of electron structures based on the Korringa-Kohn-Rostoker Green function method has been proposed. The method uses QMR to solve linear systems with multiple right hand sides. These linear systems depend on the energy-level under consideration and the convergence rate deteriorates for some energy points under consideration. While traditional preconditioners like ILU work satisfactory for the problem, the computation of the preconditioner itself is often relatively hard to parallelize. To overcome these difficulty we developed a proeconditioner that exploits the structure of the underlying heavily structured systems. The preconditioner is easy to compute and the resulting method is well-suited for the application.

THE ECKART-YOUNG THEOREM AND KY FAN'S MAXIMUM PRINCIPLE: TWO SIDES OF THE SAME COIN

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In this talk we explore the extremum properties of Orthogonal Quotients matrices. The Orthogonal Quotient Equality that we prove expresses the Frobenius norm of a difference between two matrices as a difference between the norms of two matrices. This turns the Eckart-Young minimum norm problem into an equivalent maximum norm problem. The symmetric version of this equality involves traces of matrices, and adds new insight into Ky Fan's extremum problems. A comparison of the two cases reveals a remarkable similarity between the Eckart-Young theorem and Ky Fan's maximum principle. Returning to Orthogonal Quotients matrices we derive "rectangular"extensions of Ky Fan's extremum priciples, which consider maximizing (or minimizing) sums of powers of singular values.

ON MAXIMIZATION OF SOME FUNCTIONS OF EIGENVALUE OF NONNEGATIVE DEFINITE MATRICES

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In statistical research some optimality criteria of experimental designs are formulated as functions of eigenvalues of some special class of matrices. For a given nonnegative definite matrix of order v with $0 = \lambda_0(C_d) \leq \lambda_1(C_d) \leq \ldots \leq \lambda_{v-1}(C_d)$, D-optimality criterion is formulated as the maximal value of the product of eigenvalues of C_d except $\lambda_0(C_d)$, while E-optimality criterion may be expressed as the maximal $\lambda_1(C_d)$.

For given number of ones, n, the binary matrices $T_d, L_d \in \mathbb{R}_{n\times v}$ are such that $T_d 1_v = L_d 1_v = 1_n$ and $B = I_b \otimes 1_k$, $n = bk$. The matrix C_d may be expressed as the Schur complement of $(L_d : B)^T (L_d : B)$ in the moment matrix $(T_d : L_d : B)^T (T_d : L_d : B)$. The aim of the presentation is to characterize D- and E-optimal designs for specific parameters $b = v = k$, $b = v - 2 = k - 2$ and for $L_d = (I_b \otimes H_k) T_d$, with H_k - cyclic permutation matrix. Characterization of E-optimal complete designs over some special classes of designs is given in Filipiak et al. (2008).

References

[1] Filipiak, K., R. Różański, A. Sawikowska, and D. Wojtera-Tyrakowska, *On the E-optimality of complete designs under an interference model*, *Statist. Probab. Lett.* **78**, 2470–2477, 2008.

SOME COMMENTS ON SIMILARITY, INVERSES AND SCHUR COMPLEMENTS OF GENERAL H-MATRICES[17]

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The study of properties of nonsingular M-matrices is widely used and refereed. The extension of these results to H-matrices in the invertible class [1] and the set of general H-matrices is not performed exhaustively. In the mixed class of H-matrices, any singular matrix is diagonally equivalent to its comparison matrix $\mathcal{M}(A)$ [2], a singular M-matrix; the other equimodular matrices are nonsingular H-matrices. The inverse of some of these rare H-matrices may be nonsingular M-matrix, then some of H-matrix in mix class is an inverse M-matrix. Which property may be inherited for the equimodular set or the diagonally equivalent subset? On the contrary, the inverse of some of these H-matrices be a H-matrix or not. What are the difference? Respective study on Schur complements or equimodular H-matrices in the mixed class is performed and some conclusions and questions are proposed.

References

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[17] Joint work with Rafael Bru

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IS $A \in \mathbb{C}^{n,n}$ AN H-MATRIX OR NOT?

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A summary of published and unpublished work will be presented on the identification of the $H-$ or non$-H-$matrix character of a given square complex matrix. An Algorithm will be presented which, besides the above identification, identifies also the various subclasses of $H-$ and non$-H-$matrices to which the given matrix belongs.

ON POSITIVE DEFINITENESS OF SINGULAR INTEGRAL OPERATORS [18]

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One strategy of the construction of numerical algorithms for the solution of nonlinear operator equations is based on monotonicity. In particular, if the operator is the sum of a linear operator and a nonlinear, but monotone perturbation then the positive definiteness of the linear part is of interest. We present some new results on the positive definiteness of singular operators composed by a weighted Cauchy singular operator and a multiplication operator.

[18] The talk is based on joint work with L. v. Wolfersdorf, Bergakademie Technische Universität Freiberg

COMPACT FOURIER ANALYSIS FOR MULTIGRID METHODS BASED ON THE BLOCK SYMBOL[19]

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A Compact Fourier Analysis (CFA) for designing new efficient multigrid (MG) methods is considered. The principal idea of CFA is to model MG by means of multilevel Toeplitz matrices and their generating functions and block symbols. The CFA is used for calculating smoothing factors and the total error of the combined smoothing and coarse grid correction error reduction of a twogrid step. The CFA is utilized also for deriving MG as a direct solver, i.e. an MG cycle that will converge in just one iteration step. A twogrid algorithm is considered to be a direct solver if the block symbol of the total error reduction is 0. We prove theoretical conditions that have to be fulfilled by the MG components for this purpose. General practical smoothers and transfer operators that lead to MG as direct solver are introduced. New highly efficient MG algorithms are derived by modifying appropriately the MG components that lead to the direct solver. Numerical results demonstrate the effectiveness of the theoretical approach.

[19] Joint work with Thomas Huckle

MODEL REDUCTION METHODS FOR UNSTABLE LTI SYSTEMS [20]

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The behavior of processes in many fields of applications can be described by high-dimensional systems of linear ordinary differential equations. The number of equations can easily reach a few millions. The computational cost and time needed for simulation of these processes would be extremely high if the full order models were used directly. Therefore model reduction methods that construct approximating reduced-order systems of the same form but of lower complexity are required. Most of the current methods are designed for approximating asymptotically stable linear time-invariant (LTI) systems. In this talk new interpolation-based and gramian-based methods are proposed that aim to compute a reduced-order model for stable as well as for unstable systems. In numerical experiments the accuracy of the new methods is illustrated and compared with other existing techniques for unstable systems. For systems with a large number of unstable poles the methods presented here work significantly better than techniques used so far. Moreover error bounds can be derived.

[20] Joint work with Angelika Bunse-Gerstner

ON GENERALIZATION OF THE CONCEPT OF TOTAL POSITIVITY, BASED ON THE THEORY OF CONES

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Let A be a linear operator acting in the space $\mathbb{R}^n$. Then the operator $A \wedge A$, i.e. the exterior square of the operator A, acts in the space $\wedge^2\mathbb{R}^n = \mathbb{R}^{C_n^2}$. A set $K \subset \mathbb{R}^n$ is called *a proper cone*, if it is a convex cone (i.e. for any $x, y \in K, \ \alpha \geq 0$ we have $x + y, \ \alpha x \in K$), is pointed (i.e. $K \cap (-K) = \{0\}$), closed and solid (i.e. $\mathrm{int}(K) \neq \emptyset$). A linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is called *strictly K–positive*, if there exists such a proper cone K, that $AK \subseteq \mathrm{int}(K)$. If A is strictly K-positive, then the spectral radius $\rho(A)$ is a simple positive eigenvalue of A, different in modulus from the other eigenvalues [2]. Call a linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ *strictly* $(K_1, \ K_2)$*-totally positive*, if it is strictly K-positive with respect to a proper cone $K_1 \subset \mathbb{R}^n$, and its exterior square $A \wedge A$ is strictly K-positive with respect to a proper cone $K_2 \subset \mathbb{R}^{C_n^2}$. Given two proper cones $K_1 \subset \mathbb{R}^n$ and $K_2 \subset \mathbb{R}^{C_n^2}$. Let us define the set $S(K_1, K_2) \subset \mathbb{R}^n$ by the following way:

$$S(K_1, K_2) = \overline{\{x \in \mathbb{R}^n : \exists \ x_1 \in \mathrm{int}(K_1), \ \text{for which} \ x_1 \wedge x \in \mathrm{int}(K_2)\}}.$$

If the operator A is strictly $(K_1, \ K_2)$-totally positive, then it leaves invariant the set $S(K_1, K_2)$.

Theorem 1. *Let a linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ be strictly $(K_1, \ K_2)$-totally positive. Then the operator A has 2 positive simple* *eigenvalues, different in modulus from each other and from the rest of eigenvalues:*

$$0 \leq \ldots \leq |\lambda_3| < \lambda_2 < \lambda_1.$$

Moreover, the first eigenvector x_1 *corresponding to the maximal eigenvalue* λ_1*, belongs to* $\mathrm{int}(K_1)$*, and the second eigenvector* x_2*, corresponding to the second eigenvalue* λ_2*, belongs to* $\mathrm{int}(S(K_1, K_2) \setminus K_1)$.

Theorem 1 can be easily generalized to the case of $(K_1, \ldots, K_j)$-totally positive operators, where $2 \leq j \leq n$.

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ON THE PERRON-FROBENIUS THEORY FOR COMPLEX MATRICES[21]

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We extend here the Perron-Frobenius theory of nonnegative matrices to certain complex matrices. Following the generalization of the Perron-Frobenius theory to matrices that have some negative entries, given by D. Noutsos [*Linear Algebra Appl., 412 (2006), no 2–3, 132–153*], we introduce two types of extensions of the Perron-Frobenius theory to complex matrices. We present and prove here some sufficient conditions and some necessary and sufficient conditions for a complex matrix to have a Perron-Frobenius eigenpair. We apply this theory by introducing the Perron-Frobenius splittings, as well as the complex Perron-Frobenius splittings, for the solution of the complex linear systems $A\mathbf{x} = \mathbf{b}$ by classical iterative methods. Perron-Frobenius splittings constitute an extension of the well-known regular splittings, weak regular splittings and nonnegative splittings. Convergence and comparison properties are also given and proved.

[21] Joint work with Richard S. Varga

ITERATIVE METHOD FOR COMPUTING MOORE-PENROSE INVERSE BASED ON PENROSE EQUATIONS[22]

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We develop an iterative algorithm for estimating the Moore-Penrose generalized inverse. The main motive for the construction of algorithm is simultaneous usage of Penrose equations (2) and (4). Convergence properties of the introduced method are considered as well as their first-order and the second-order error terms. Numerical experience is also presented.

[22]Joint work with Predrag S. Stanimirović

ON KACZMARZ'S PROJECTION ITERATION AS A DIRECT SOLVER FOR LINEAR LEAST SQUARES PROBLEMS[23]

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In the last 30 years, a large class of direct projection-based methods has been developed for numerical solution of large and sparse linear systems of equations (see [1] and references therein). They are essentially using projections onto the hyperplanes generated by the system equations, following some directions which are constructed according to different principles and purposes (see [1] and references therein). In the present paper we develop a similar method starting from Kaczmarz's projection algorithm and its extension to linear least squares problems (see [2] and references therein). By writing Kaczmarz's algorithm as a classical iteration $x^{k+1} = Qx^k + c$, we first observe that if the iteration matrix vanishes on all the system rows, i.e. $QA_i = 0, \forall i (*)$ then the first constructed approximation is already a solution, i.e. Kaczmarz's iteration becomes a direct solver. We design an algorithm for a recursive generation of new directions for projection (i.e. new equations in the initial system) such that, for the extended system the vanishing relations (*) hold. This algorithm is then adapted to the extended Kaczmarz method from [2] such that a direct projection-based solver for general linear least squares problems is obtained.

[23] Joint work with Harald Koestler, Tobias Preclik and Ulrich Ruede

Numerical experiments with the above mentioned algorithms, and comparisons with some versions of the DPM ones from [1] have been performed on large sparse linear systems of equations arising from multibody dynamics problems.

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SPLIT BEZOUTIANS AND INVERSES OF SYMMETRIC TOEPLITZ OR CENTROSYMMETRIC TOEPLITZ-PLUS-HANKEL MATRICES

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Inverses of symmetric Toeplitz or centrosymmetric Toeplitz-plus-Hankel matrices can be represented as a sum of two split Bezoutians, one of (+)-type, the other of (−)-type. Ideas how to develop such inversion formulas are represented. Moreover, matrix representations for split Bezoutians are discussed.

A STABLE VARIANT OF SIMPLER GMRES AND GCR [24]

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Minimum residual Krylov subspace methods form a popular class of iterative methods for solving large and sparse nonsymmetric systems of linear algebraic equations. Besides the GMRES method [5], other mathematically equivalent implementations like Simpler GMRES [6] and namely GCR [1] are used sometimes in practice. As shown in [6, 4, 3] their numerical behavior depends strongly on the conditioning of the basis of the Krylov subspace, which appears to be directly linked to the convergence of the residual norms. While the condition number of the basis used in Simpler GMRES is growing with decreasing relative residual norms, fast convergence of the residual norms results in the well-conditioned residual basis of GCR. We propose a stable variant of Simpler GMRES and GCR [2], which is based on the adaptive choice of the Krylov subspace basis at a given iteration step using the intermediate residual norm decrease criterion. The new direction vector is chosen as in the original implementation of Simpler GMRES or it is equal the normalized residual vector as in the GCR method. Such an adaptive strategy leads to a well-conditioned basis of the Krylov subspace, which provides a numerically stable and more robust variant of Simpler GMRES or GCR.

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[24] Joint work with Pavel Jiranek

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MAX-ALGEBRAIC POWERS OF NONNEGATIVE MATRICES

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Max algebra is interesting analogue of nonegative linear algebra, where the ordinary addition is replaced by max. We consider max algebraic powers of nonnegative matrices, both in irreducible and reducible case. The main idea is to apply an appropriate diagonal similarity scaling, after which the connections with Boolean matrix algebra can be exploited. In the irreducible case, we describe cones of vectors which converge to an eigenvector, and in the reducible case we obtain CSR expansions of matrix powers and investigate the reachability problem.

ON GENERALIZED EVEN AND ODD OSCILLATORY OPERATORS[25]

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Let A be a linear operator acting in the space $\mathbb{R}^n$. In this case we can define operators $\otimes^j A$ and $\wedge^j A$ $(j = 1, \ldots, n)$, i.e. the j-th tensor and the j-th exterior power of the operator A. They acts, respectively, in the space $\otimes^j \mathbb{R}^n = \mathbb{R}^{n^j}$ and $\wedge^j \mathbb{R}^n = \mathbb{R}^{C_n^j}$. Let $\{\lambda_i\}_{i=1}^n$ be all eigenvalues of the operator A, repeated according to multiplicity. Then all the possible products of the type $\{\lambda_{i_1} \ldots \lambda_{i_j}\}$, where $1 \leq i_1 < \ldots < i_j \leq n$, form all the possible eigenvalues of the exterior power $\wedge^j A$, repeated according to multiplicity [3]. A set $K \subset \mathbb{R}^n$ is called *a proper cone*, if it is a convex cone (i.e. for any $x, y \in K$, $\alpha \geq 0$ we have $x + y$, $\alpha x \in K$), is pointed (i.e. $K \cap (-K) = \{0\}$), closed and solid (i.e. $\mathrm{int}(K) \neq \emptyset$) [2]. A linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is called *K–primitive*, if there exists such a proper cone K, that $AK \subseteq K$ and the only nonempty subset of $\partial(K)$ which is left invariant by A is $\{0\}$. Let a linear operator A be K-primitive. Then the spectral radius $\rho(A)$ is a simple positive eigenvalue of A, different in modulus from the other eigenvalues [3]. A linear operator A is called *generalized oscillatory* if it is K-primitive with respect to a proper cone $K_1 \subset \mathbb{R}^n$, and for every j $(1 < j \leq n)$ its j-th exterior power $\wedge^j A$ is K-primitive with respect to a proper cone $K_j \subset \mathbb{R}^{C_n^j}$.

[25] Joint work with O. Y. Kushel

Theorem 1. *Let a linear operator* $A : \mathbb{R}^n \to \mathbb{R}^n$ *be generalized oscillatory. Then all the eigenvalues of the operator* A *are simple, positive and different in modulus from each other:*

$$\rho(A) = \lambda_1 > \lambda_2 > \ldots > \lambda_n > 0.$$

A linear operator A is called *generalized even (odd) oscillatory* if for every even (respectively odd) j $(1 \leq j \leq n)$ its j-th exterior power $\wedge^j A$ is K-primitive with respect to a proper cone $K_j \subset \mathbb{R}^{C_n^j}$.

Theorem 2. *Let a linear operator* $A : \mathbb{R}^n \to \mathbb{R}^n$ *be even generalized oscillatory. Then the algebraic multiplicity* $m(\lambda)$ *of any eigenvalue* λ *of the operator* A *is not greater than* 2*. The following inequalities for the modules of the eigenvalues are true:*

$$\rho(A) = |\lambda_1| \leq |\lambda_2| < |\lambda_3| \leq |\lambda_4| < \ldots.$$

(The eigenvalues of A *are repeated according to multiplicity in the above numeration.) Moreover, for every pair* $\lambda_i \lambda_{i+1}$ $(i = 1,\ 3,\ 5,\ \ldots)$ *the following equality is true:* $\arg(\lambda_{i+1}) = -arg(\lambda_i)$*. If* n *is odd, then* λ_n *is real.*

Theorem 3. *Let a linear operator* $A : \mathbb{R}^n \to \mathbb{R}^n$ *be odd generalized oscillatory. Then the algebraic multiplicity* $m(\lambda)$ *of any eigenvalue* λ *of the operator* A *is not greater than* 2*. The following inequalities for the modules of the eigenvalues are true:*

$$\rho(A) = |\lambda_1| < |\lambda_2| \leq |\lambda_3| < |\lambda_4| \leq \ldots.$$

(The eigenvalues of A *are repeated according to multiplicity in the above numeration.) Moreover,* $\lambda_1 = \rho(A)$ *is a simple positive eigenvalue of* A*. If* n *is even, then* λ_n *is real. For every pair* $\lambda_i \lambda_{i+1}$ $(i = 2,\ 4,\ 6,\ \ldots)$ *the following equality is true:* $\arg(\lambda_{i+1}) = -arg(\lambda_i)$*.*

Theorems 1, 2 and 3 can be easily reformulated in terms of compound matrices. Then the conditions of the theorems can easily verified.

TRACE INEQUALITIES FOR MATRICES [26]

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In this paper we present trace inequalities for matrices, some these inequalities improve a result of Chen and Wong, generalize results of Manjegani and answer a conjecture of Xinmin Yang in the case of matrices. Amoung other inequalities it is shown that, if A and B are an nn complex matrices then $tr(AB)^k \leq \min(||A|||^k trB^k, ||B||^k trA^k)$. Related inequalities are also presented.

[26] Joint work with Hussien Albadawi

MATRIX VERSION OF THE CHEBYSHEV AND KANTOROVICH INEQUALITIES[27]

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The Chebyshev inequality states that

$$\left(\sum_{i=1}^{n} w_i a_i\right)\left(\sum_{i=1}^{n} w_i b_i\right) \leq \left(\sum_{i=1}^{n} w_i\right)\left(\sum_{i=1}^{n} w_i a_i b_i\right) \tag{5}$$

for all non-negative real numbers $a_1 \geq a_2 \geq \cdots \geq a_n$, $b_1 \geq b_2 \geq \cdots \geq b_n$ and weights $w_i \geq 0,\ i = 1, \ldots, n$. If $0 < a \leq a_i \leq b,\ w_i \geq 0,\ i = 1, 2, \ldots, n$, the Kantorovich's inequality states that

$$\left(\sum_{i=1}^{n} w_i a_i\right)\left(\sum_{i=1}^{n} \frac{w_i}{a_i}\right) \leq \frac{(a+b)^2}{4ab}\left(\sum_{i=1}^{n} w_i\right)^2. \tag{6}$$

We prove matrix versions of the above inequalities involving the Hadamard product i.e. entrywise product of the matrices. More specifically we prove that

$$\left(\sum_{i=1}^{n} w_i A_i\right) \circ \left(\sum_{i=1}^{n} w_i B_i\right) \leq \left(\sum_{i=1}^{n} w_i\right)\left(\sum_{i=1}^{n} w_i (A_i \circ B_i)\right) \tag{7}$$

for $n \times n$ positive semidefinite matrices $A_i, B_i, i = 1, \ldots, n$, such that $A_1 \geq \cdots \geq A_n,\ B_1 \geq \cdots \geq B_n$ where $w_i \geq 0,\ i = 1, \ldots, n$, are weights.

[27] Joint wok with Jagjit Singh Matharu and Jaspal Singh Aujla

ON CHEBYSHEV POLYNOMIALS OF MATRICES[28]

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The mth Chebyshev polynomial of a square matrix A is the monic polynomial that minimizes the matrix 2-norm of $p(A)$ over all monic polynomials $p(z)$ of degree m. This polynomial is uniquely defined if m is less than the degree of the minimal polynomial of A. In this talk we study general properties of Chebyshev polynomials of matrices, which in some cases turn out to be generalizations of well known properties of Chebyshev polynomials of compact sets in the complex plane. We also derive explicit formulas of the Chebyshev polynomials of certain classes of matrices.

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[28] Joint work with Vance Faber and Jörg Liesen

LAVEL-BASED ALGEBRAIC PRECONDITIONING[29]

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Algebraic preconditioning algorithms have achieved throughout the time a considerable degree of efficiency and robustness. In spite of this, solving problems from many applications require tools to further enhance convergence of preconditioned iterative methods and to provide useful results with increased reliability.

In this talk we deal with the level-based preconditioning which represents one of two basic approaches for algebraic preconditioning of iterative methods. This strategy can be sometimes very useful but it may suffer from excessive memory demands. Here we present its improvements which consider a non-uniform setting of levels to the entries of the underlying graph from which the levels are derived. This non-uniform setting is based on the numerical information extracted from the matrix. Based on the extensive experiments we show that this new level-based preconditioning is more efficient than the standard one. In particular, it is demonstrated, that our new level-based option may significantly extend the spectrum of sizes for which this algebraic preconditioner provides a converging iterative method.

[29]This work was supported by the international collaboration support M100300902 of AS CR.
Joint work with Jennifer Scott

Applied Linear Algebra - Short Communications

CONNECTED GRAPHS OR FIXED ORDERED AND SIZE WITH MAXIMAL Q-INDEX: SOME SPECTRAL BOUNDS[30]

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The Q-index (or spectral radius) of a simple graph is the largest eigenvalue of its signless Laplacian matrix. There are many results in the literature where, for some fixed classes of graphs, all graphes whose index is maximal are identified. For connected graphs of fixed order and size this problem is not yet completely resolved (in contrast to the more general class when connectivity is not required). It is only known (for a long time) that the graphs with maximal Q-index in the former class (and the latter one) are the nested split graphs. In this paper we focusing our attention on eigenvector techniques for getting some new (lower and upper) bounds on the Q-index of nested split graphs.

[30] Joint work with Slobodan Simić, C. M. de Fonseca and Dejan V. Tošić

THE GRAM MATRIX IN INNER PRODUCT MODULES OVER C^*-ALGEBRAS[31]

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Let $(X, \langle\cdot,\cdot\rangle)$ be a semi-inner product module over a C^*-algebra A. It is known that, for an arbitrary $n \in \mathbb{N}$ and $x_1, \ldots, x_n \in X$, the Gram matrix $[\langle x_i, x_j\rangle]$ is a positive element of the matrix algebra $M_n(A)$. We show, by defining a suitable new semi-inner product on X, that a stronger inequality, namely $[\langle x_i, x_j\rangle] \geq \frac{1}{\|z\|^2}[\langle x_i, z\rangle\langle z, x_j\rangle]$, holds true for all $z \in X$ such that $\langle z, z\rangle \neq 0$. By the same technique, we show that the inequality $[\langle x_i, x_j\rangle] \geq \frac{1}{\|z\|^2}[\langle x_i, z\rangle\langle z, x_j\rangle]$ can be refined by a sequence of nested inequalities.

This leads to some interesting operator-theoretical consequences. In particular, we construct, for an invertible positive Hilbert space operator a, a sequence that converges in norm to a^{-1}. When applied to a positive invertible matrix, this gives us an algorithm for computing a^{-1} in which the number of steps does not exceed the number of elements of the spectrum of a.

[31] Joint work with D. Bakić and M.S. Moslehian

FULL RANK FACTORIZATIONS[32]

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In this lecture we present some full rank factorizations of matrices, such that the full rank Cholesky factorization of a singular symmetric matrix, the thin QR factorization of rectangular matrices, the Jordan form of a singular square matrix and the full rank SVD of a rectangular matrix.

[32] Joint work with Beatriz Ricarte and Ana M. Urbano

ON THE STRUCTURE OF SPLIT LIE ALGEBRAS

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A splitting Cartan subalgebra H of a Lie algebra L is defined as a maximal abelian subalgebra of L satisfying that the adjoint mappings $ad(h)$ for $h \in H$ are simultaneously diagonalizable. If L contains a splitting Cartan subalgebra H, then L is called a split Lie algebra. This means that we have a root decomposition $L = H + (\sum_{\alpha \in \Lambda} L_\alpha)$ where $L_\alpha = \{v_\alpha \in L : [h, v_\alpha] = \alpha(h)v_\alpha \text{ for any } h \in H\}$ for a linear functional $\alpha \in H^*$ and $\Lambda := \{\alpha \in H^* \backslash \{0\} : L_\alpha \neq 0\}$ is the corresponding root system. The subspaces L_α for $\alpha \in H^*$ are called root spaces of L (respect to H) and the elements $\alpha \in \Lambda \cup \{0\}$ are called roots of L respect to H. We develop techniques of connections of roots for split Lie algebras with symmetric root systems. We show that any of such algebras L is of the form $L = \mathcal{U} + \sum_j I_j$ with $\mathcal{U}$ a subspace of the abelian Lie algebra H and any I_j a well described ideal of L, satisfying $[I_j, I_k] = 0$ if $j \neq k$. Under certain conditions, the simplicity of L is characterized and it is shown that L is the direct sum of the family of its minimal ideals, each one being a simple split Lie algebra with a symmetric root system and having all its nonzero roots connected.

STUDY OF LIE ALGEBRAS BY USING COMBINATORIAL STRUCTURES[33]

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This paper establishes a link between Lie algebras and Graph Theory: specific combinatorial structures of dimension 2 can be associated with the Lie algebra formed by upper-triangular (resp. strictly upper-triangular) matrices. Such algebras are used to represent solvable (resp. nilpotent) Lie algebras. Moreover, the converse problem is also developed in this paper by computing the families of n-dimensional Lie algebras associated with specific combinatorial structures and graphs made up of n vertices.

[33] Joint work with Juan Núñez and Ángel F. Tenorio

INCREMENTAL CONDITION ESTIMATION WITH INVERSE TRIANGULAR FACTORS[34]

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A popular procedure for condition estimation was proposed in a 1990 paper by Bischof. The approximate condition numbers of the factors of a triangular decomposition are obtained in an incremental way from the principal submatrices. The approximation for the current principal submatrix relies on an approximate left singular vector constructed without accessing the columns or rows of smaller principal submatrices. This makes the procedure relatively inexpensive and particularly suited when a triangular matrix is computed one column or row at a time. In 2002, Duff and Vömel came up with a similar strategy based on approximate *right* singular vectors and recommended its use for sparse matrices. In this talk we will demonstrate and give a theoretical explanation for the fact that the second technique, based on right singular vectors, gives in general better estimates of the maximal singular value of dense matrices. Furthermore, we show how this can be exploited when the inverse of the triangular matrix is available. In this case we obtain an incremental condition estimator which is significantly better than either the original or the modified 2002 technique.

[34] Joint work with Miroslav Tuma

ON AN EFFICIENT FAMILY OF SIMULTANEOUS METHODS FOR FINDING POLYNOMIAL ZEROS[35]

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A family of iterative methods for the simultaneous determination of simple zeros of algebraic polynomials is stated. This family is more efficient compared to all existing simultaneous methods based on fixed point relations. A very high computational efficiency is obtained using suitable corrections resulting from a family of two-point fourth order methods of low computational complexity. The presented convergence analysis shows that the convergence rate of the basic third order method is increased from 3 to 6 using this special type of corrections and applying only n additional polynomial evaluations per iteration. Some computational aspects and numerical examples are given to demonstrate very fast convergence and high computational efficiency of the proposed zero-finding family.

[35] Joint work with Miodrag S. Petković and Ljiljana D. Petković

MAPS PRESERVING THE EIGENVALUE INCLUSION REGIONS[36]

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Let $G(A)$ be the Gershgorin region of a square matrix A. Characterization is obtained for maps f on matrices such that $G(AB) = G(f(A)f(B))$ for every pair of n-by-n matrices A and B. Similar results are proved when the set $G(A)$ is replaced by other eigenvalue inclusion regions such as the Brauer set or the Ostrowski set on square matrices.

[36] Joint work with Aaron Herman, Chi-Kwong Li, Nung-Sing Sze, Vincent Yannello

FINITELY CENTRALLY GENERATED C*-ALGEBRAS

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We say that a C*-algebra A is *finitely centrally generated* if A as a module over the center of its multiplier algebra is finitely generated. We show that A is finitely centrally generated if and only if A is a finite direct sum of unital homogeneous C*-algebras (a C*-algebra B is called *n-homogeneous* if all irreducible representations of B are of the same finite dimension n. It is well known that such B is naturally isomorphic to the C*-algebra $\Gamma_0(E)$ of all continuous sections which vanish at infinity of the underlying (locally trivial) $\mathrm{M}_n(\mathbb{C})$-bundle E over $\mathrm{Prim}(B)$).

THE ENERGY OF INTEGRAL CIRCULANT GRAPHS [37]

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Circulant graphs are an important class of interconnection networks in parallel and distributed computing. Integral circulant graphs play an important role in modeling quantum spin networks supporting the perfect state transfer as well. The integral circulant graph $ICG_n(D)$ has the vertex set $Z_n = \{0, 1, 2, \ldots, n-1\}$ and vertices a and b are adjacent if $\gcd(a-b, n) \in D$, where $D \subseteq \{d : d \mid n,\ 1 \leq d < n\}$. These graphs are highly symmetric, have integral spectra and some remarkable properties connecting chemical graph theory and number theory.

The energy of G was first defined by Gutman, as the sum of the absolute values of eigenvalues of the adjacency matrix. Recently, some other energy-based graph invariants were introduced, such as distance and Laplacian energy. There was a vast research for the pairs and families of non-cospectral graphs having equal energies. Pairs of integral circulant graphs are constructed, having equal energy, Laplacian energy and distance energy. Furthermore, for every fixed $k \in N$, families of k hyperenergetic non-cospectral integral circulant n-vertex graphs with equal energy are presented.

We establish some general closed form expressions for the energy of integral circulant graphs when the set of divisors D has one or two

[37] Joint work with Milan Bašić

elements or when n is a square-free number. Especially, we consider the energy of integral circulant graphs for $1, n/2 \in D$, which is important for the characterization of $E(ICG_n(D))$ modulo 4.

GENERALIZED BICIRCULAR PROJECTIONS ON SOME MATRIX AND OPERATOR SPACES [38]

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Let X be a complex Banach space and let $P : X \to X$ be a linear projection, that is a linear mapping with the property $P^2 = P$. By $\overline{P}$ is denoted its complementary projection $I - P$, where I is the identity operator on X. A projection P is called a generalized bicircular projection if the mapping $P + \lambda\overline{P}$ is an isometry for some modulus one complex number $\lambda \neq 1$. The aim of this talk is to describe the structure of these mappings on $S(H)$ and $A(H)$, the linear subspace of all symmetric operators on a complex Hilbert space H, and the linear subspace of all antisymmetric operators on H, respectively. Both the case when H is finite dimensional and the case when H is infinite dimensional will be discussed.

[38] Joint work with Ajda Fošner, Maja Fošner and Chi-Kwong Li

FINITE DIFFERENCE SCHEME FOR A TWO-DIMENSIONAL PARABOLIC-HYPERBOLIC TRANSMISSION PROBLEM[39]

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An initial boundary value problem for a two-dimensional parabolic-hyperbolic equation in two disjoint rectangles is investigated. The effect of the intermediate layer on the solution is modeled by means of nonlocal jump conditions across the interface. Existence and uniqueness of weak solutions are proved and a priori estimates in appropriate Sobolev-like spaces are derived. Few finite difference schemes approximating this problem are proposed and analyzed. Estimates of the convergence rate compatible with the smoothness of the input data are obtained.

[39] Joint work with Lubin Vulkov

BOOTSTRAP AMG FOR MARKOV CHAIN COMPUTATIONS[40]

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Markov Chains play an important role to model various stochastic processes in many different fields. In most cases one is interested in the computation of steady states of these processes, i.e., the eigenvectors corresponding to the eigenvalue one. Several approaches using Multigrid techniques or Krylov subspace methods were proposed over the years to efficiently compute steady state vectors in cases, where simple iterative methods (e.g., power method) are slow to converge. We develop a novel adaptive Bootstrap algebraic Multigrid method for the computation of steady state vectors of slowly mixing Markov Chains and analyze its application as a preconditioner for suitable Krylov subspace methods.

[40] Joint work with James Brannick, Ira Livshits, Achi Brandt, Andreas Frommer and Matthias Bolten

ON UNICYCLIC REFLEXIVE GRAPHS[41]

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A simple graph is reflexive if the second largest eigenvalue of its $(0,1)-$ adjacency matrix does not exceed 2. A vertex of the cycle of unicyclic simple graph is said to be loaded if its degree is greater than 2. The maximum number of loaded vertices of the cycle in unicyclic reflexive graph is 8 and it is attained only in the case of octagon. In this paper we examine the interrelation between the number of loaded vertices of the cycle and its length in unicyclic reflexive graph, including in some cases also the influence of the disposition of loaded vertices. Among others, we establish that the maximum length of the cycle with 7 loaded vertices is 10, with 6 loaded vertices is 12, with 5 loaded vertices is 16, while in case of less than 5 loaded vertices this length is not limited, but it becomes limited on some additional conditions.

[41] Joint work with Zoran Radosavljević

SOLVING SECOND ORDER DISCRETE STURM-LIOUVILLE BVP USING MATRIX PENCILS[42]

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This paper deals with discrete second order Sturm-Liouville Boundary Value Problems (DSLBVP) where the parameter that as part of the Sturm-Liouville difference equation appears nonlinearly in the boundary conditions. We focus on analyzing SLBVP with cubic nonlinearity in the boundary condition. The problem is described by a matrix equation with nonlinear variables. By applying the matrix pencil techniques, we reduce an SLBVP to a generalized eigenvalue problem with respect to its coefficient matrix. Under certain conditions, such a generalized eigenvalue problems can be further reduced to a regular eigenvalue problem so that many existing computational tools can be applied to solve the problem. The main results of the paper provide the reduction procedure and identify those cubic SLBVPs which can be reduced to regular eigenvalue problems. We also investigate the structure of the coefficient matrix to a DSLBVP and its effect on the reality of the eigenvalues of the problem.

[42] Joint work with Michael Wilson

FEW SUBPOWERS AND GENERALIZED GAUSSIAN ELIMINATION[43]

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We present a problem of computational complexity of Čonstraint Satisfaction Problem"which has been under heavy scrutiny by researchers from various areas of mathematics and computer science in the last decade. In this talk we will review an algebraic property of the template for the Čonstraint Satisfaction Problem"which characterizes the templates for which a generalization of Gaussian elimination called the Bulatov-Dalmau algorithm can be employed to solve the Constraint Satisfaction Problem in deterministic polynomial time (this result was obtained by the team of authors from the title). The class of problems under review presents one of the largest two known for which a polynomial time algorithm for solving fixed-template Constraint Satisfaction Problem has been proved to exist.

[43]Joint work with Joel Berman, Pawel Idziak, Ralph McKenzie, Matthew Valeriote and Ross Willard

ISM PRECONDITIONERS FOR NONSYMMETRIC MATRICES

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We study the Inverse Shermann-Morrison decomposition and how it can be used to compute preconditioners. We analyze the relation of the ISM and LDU factorizations and how this relation can be exploited to apply smart dropping strategies. The influence of the factor s of the ISM factorization is also studied. Several numerical examples are showed.

REFLEXIVITY IN SOME CLASSES OF MULTYCYCLIC TREELIKE GRPHS [44]

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A simple graph is treelike if every two cycles of that graph have at most one common vertex. A graph is reflexive if the second largest eigenvalue of its $(0,1)-$ adjacency matrix does not exceed 2. We consider graphs in which removing one of its vertices does not determine reflexivity and whose cycles does not form a bundle. The known result is that such graphs have at most five cycles. We present our prior results, current investigations and results and open problems in described classes.

[44]Joint work with Zoran Radosavljević and Marija Račajski

THE STUDY OF INVARIANTS OF HOMOGENOUS MATRIX PENCILS $sF - \hat{s}G$ UNDER STRICT EQUIVALENCE[45]

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In linear system and control theory, generalized time invariant systems are intimately related to matrix pencil theory. Actually, a large number of systems are reduced to the study of differential/difference systems $S(F,G)$ of the form: $S(F,G) : F\dot{\underline{x}}(t) = G\underline{x}(t)$ / $F\dot{\underline{x}}_{k+1} = G\underline{x}_k$, $F, G \in C^{m\times n}$ and their properties can be characterized by the homogeneous pencil $sF - \hat{s}G$. An essential problem in matrix pencil theory is the study of invariant of $sF - \hat{s}G$ under the bilinear strict equivalence. This problem is equivalent to the study of complete Projective Equivalence (PE), $\mathcal{E}_p$, defined on the set C_r of complex homogeneous binary polynomials of fixed homogeneous degree r. For an $f(s,\hat{s}) \in C_r$, the study of invariants of the PE class $\mathcal{E}_p$ is reduced to a study of invariants of matrices of the set $C^{l\times 2}$ ($l \times 2$ matrices, for $l \geq 3$ with all 2×2 minors non-zero) under the Extended Hermite Equivalence (EHE), $\mathcal{E}_{rH}$. In this paper, we present some interesting properties of the PE and for the EHE class. Moreover, we calculate analytically the appropriate transformation $b \in RGL(1, C/R)$.

[45] Joint work with Athanasios Karageorgos and Grigoris Kalogeropoulos

COMPARISON BETWEEN SOME MATRIX METHODS WITH APPLICATION IN PATTERN RECOGNITION

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In this paper we present the comparison for three approaches to investigate the pattern recognition: the eigenfaces technique, the HOSVD one (Higher Order Singular Value Decomposition) for tensors (along with a low-rank SVD approximation that computes the truncation level and that reduces the running time of the algorithms), and finally, the nonnegative matrix factorization (NMF) one. It is known that all NMF algorithms are iterative and they are sensitive to the initialization of the involved matrices W and H; in all cases, a good initialization can improve the speed and the accuracy of the algorithms, as it can produce faster convergence to an improved local minimum, and nearly all NMF algorithms use simple random initialization. Therefore, we shall present some cases of random initialization. We conclude the paper by presenting some examples with pattern recognition, and our conclusions regarding the use of these approaches, both for faces and digits database.

A NUMERICAL RANGE FOR RECTANGULAR MATRIX POLYNOMIALS[46]

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The numerical range of an operator can be written as an infinite intersection of closed circular discs. This interesting property was observed by Bonsall and Duncan (1973), and leads (in a natural way) to a definition of numerical range of rectangular complex matrices. The new range is always compact and convex, and satisfies basic properties of the standard numerical range. The proposed definition is also extended to the case of matrix polynomials.

[46]joint work with Christos Chorianopoulos

COMPANION FACTORIZATION IN THE GENERAL GROUP $GL(n;\mathbb{C})$ AND APLICATIONS[47]

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A constructive analysis on linear area-preserving maps has been recently introduced using a factorization for the associated shift matrices as product of first companion matrices. Now, the companion factorization is extended to the general linear group of regular complex matrices with any order n. The elements of first companion matrices are explicitly represented in terms of determinants of submatrices of the matrix being factorized. This permits us to extend to general linear complex maps our constructive analysis based on difference equations. A remarkable application is the representation of both the product of k arbitrary first companion matrices and the inverse of any regular matrix, for which an inverse factorization is straightforward. In addition, this companion factorization could be of interest in the non-numeric branches of Linear Algebra as well as in symbolic computation.

[47] Joint work with J. Abderramán Marrero

ON THE OPERATIONAL SOLUTION OF THE SYSTEM OF FRACTIONAL DIFFERENTIAL EQUATIONS

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We consider the linear system of differential equations with the fractional derivatives, and its corresponding system in the field of Mikusinski operators, , written in a matrix form. First, the matrices with operator entries are analyzed, and then, the exact operational solution of the corresponding matrix equations is determined, by using the connection between the fractional and the Mikusinski calculus. The form of the operational approximate solution is constructed, and the conditions for its existence are obtained. The character of the exact and approximate operational solutions is analyzed, and the error of approximation is estimated.

SPECTRAL STUDY OF GENERALIZED POTENT MATRICES [48]

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The idempotent matrices have been used in many applications in different areas. In order to generalize the potent matrices [1], we will analyze a new kind of matrices. On the other hand, it is well-known that spectral theory of matrices is an important approach to obtain information about the behavior of the matrix. This class of matrices will be studied from a spectral point of view. Moreover, in recent years, linear combinations of two matrices that satisfy certain properties about their powers have been widely studied [2]. Related to this new class of matrices, we will design an algorithm to compute all the possible linear combinations in the same class.

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[2] J. Benítez, N. Thome, *$\{k\}$-group periodic matrices*, *SIAM J. Matrix Anal. Appl.* 28, 1, 9–25, 2006.

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Joit work with Leila Lebtahi, Óscar Romero

DIMENSION REDUCTION FOR DAMPING OPTIMIZATION IN LINEAR VIBRATING SYSTEMS [49]

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We will consider a mathematical model of a linear vibrational system $M\ddot{x} + D\dot{x} + Kx = 0$, where M and K are mass and stiffness, respectively, which are positive definite matrices of order n. Damping matrix is $D = C_u + C$, where C_u represents internal damping and C is external damping which is positive semidefinite matrix. Problem of determination of optimal damping is equivalent to minimization of the trace of the solution of the Lyapunov equation

$$AX + XA^T = -Z\,,$$

where A is matrix $2n \times 2n$ obtained from M, D and K. We are interested in the case where $Z = I$, which corresponds to the case when all eigenfrequencies of the undamped system are damped. Finding the optimal C such that the trace of $X(C)$ is minimal is a very demanding problem, caused by the large number of trace calculations, which are required for bigger matrix dimensions. Thus we propose dimension reduction to accelerate optimization process. We will present an approximation of the solution of the structured Lyapunov equation and a corresponding error bound for the approximation. Our algorithm

[49] Joint work with Peter Benner and Ninoslav Truhar

for efficient approximation of the optimal damping is based on this approximation.

ON BOUNDARY DISCRETE MAXIMUM PRINCIPLE IN THE FINITE ELEMENT METHOD

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In this contribution we consider a finite element approximation of a general linear elliptic problem

$$\begin{aligned} -\operatorname{div}(\mathcal{A}\nabla u) + b \cdot \nabla u + cu &= f \quad \text{in } \Omega, \\ u &= g_D \quad \text{on } \Gamma_D, \\ \alpha u + (\mathcal{A}\nabla u)\cdot n &= g_N \quad \text{on } \Gamma_N, \end{aligned}$$

and ask a question whether nonnegative data ($f \geq 0$, $g_D \geq 0$, $g_N \geq 0$) imply nonnegative finite element solution ($u_h \geq 0$). This is known as the problem of the discrete maximum principle (DMP). The validity of the DMP can be characterized by the nonnegativity of the corresponding discrete Green's function (DGF). In the case of nonvanishing Dirichlet boundary data g_D there are actually two DGFs – the interior and the boundary one. In the talk we will present a general approach how to express the boundary DGF. In the special case of the lowest-order (e.g. linear) finite elements this approach leads to sufficient and necessary conditions for the validity of the DMP in terms of nonnegativity of certain matrices.

A PERMUTED FACTORS APPROACH FOR THE LINEARIZATION OF POLYNOMIAL MATRICES[50]

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In recent works of the authors, a new family of companion forms associated to a regular polynomial matrix $T(s)$ has been presented, using products of permutations of n elementary matrices, generalizing similar results by M. Fiedler where the scalar case was considered. In this paper, extending this permuted factors approach, we present a broader family of companion like linearizations, using products of up to $n(n-1)/2$ elementary matrices, where n is the degree of the polynomial matrix. Under given conditions, the proposed linearizations can be shown to consist of block elements where the coefficients of the polynomial matrix appear intact. Additionaly we provide a criterion for those linearizations to be block symmetric. We also illustrate several new block symmetric linearizations strictly equivalent to the original polynomial matrix $T(s)$ where in some of them, the constraint of nonsingularity of the constant term and the coefficient of maximum degree is not a prerequisite.

[50] Joint work with Efstathios Antoniou

STABILITY OF FINITE-DIFFERENCE DISCRETIZATIONS OF SINGULAR PERTURBATION PROBLEMS

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In this paper, a survey of various stability results is given for finite-difference schemes used to discretize singular perturbation problems. For this purpose, it suffices to consider the following simplified problem:

$$Lu := -\varepsilon u'' - b(x)u' + c(x)u = 0, \quad x \in [0,1], \quad u(0) = U_0, \quad u(1) = U_1, \tag{8}$$

where ε is a small positive perturbation parameter, b and c are sufficiently smooth functions, and U_0 and U_1 are given constants. The problem is to find a $C^2[0,1]$-solution u of problem (**??**). When this is done numerically, it is important to obtain errors which decrease uniformly in ε as the discretization parameter N (the number of mesh subintervals) tends to ∞. This is known as convergence uniform in ε and requires special numerical methods, which typically have to be both consistent and stable uniformly in ε. The methods considered here use finite-difference schemes on special non-equidistant meshes of Bakhvalov and Shishkin types. When problem (**??**) is discretized, a system of linear equations is obtained. If A is the matrix of this system, stability uniform in ε means that A is nonsingular and that, in some suitable matrix norm, $\|A^{-1}\| \leq M$, where M is a positive constant independent of ε and N. Stability uniform in ε

can be proved in different ways, depending on the problem and the discretization scheme. In the simplest cases, the proof is based either on strict diagonal dominance or on M-matrices (inverse-monotone L-matrices). As schemes become more complicated for the purpose of achieving a greater accuracy, less trivial methods of proof have to be used, like different appropriate decompositions of the matrix. For instance, Lorenz's "standard-decomposition" is used to prove inverse monotonicity in the absence of an L-matrix. Also, the smallness of ε may have to be invoked. All these techniques are illustrated by considering discretizations of different types of problem (**??**).

MAPS PRESERVING THE PSEUDO SPECTRUM[51]

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Let $\sigma_\varepsilon(A)$ be the pseudo spectrum of a square matrix A. Characterization is obtained for maps f on matrices such that $\sigma_\varepsilon(AB) = \sigma_\varepsilon(f(A)f(B))$ for every pair of n-by-n matrices A and B. Extension of the result will be discussed.

[51] Joint work with Jianlian Cui, Chi-Kwong Li, Vincent Yannello

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